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ON THE SHOCK STAND-OFF DISTANCE IN AN INVISCID HYPERSONIC SOURCE FLOW PAST TWO-DIMENSIONAL BLUFF BODIES

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AIR FORCE SYSTEMS COMMAND
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AEROSPACE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This report was prepared by the Hypersonic Research Laboratory of the Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, under Project 7064 entitled, "High Velocity Fluid Mechanics."

The results given in this report are a by-product of a study of the effects of Mach number gradients prevalent in wedge-shaped or conical nozzles employed for hypersonic wind tunnel testing.

ABSTRACT

The effect of high Mach number flows on test bodies can be estimated reasonably well by using the Newtonian approximation. If the flow is like a source flow then the effects of the ambient flow gradients on the flow parameters in the shock layer can be obtained by modifying the Newtonian formula. Freeman's theory is adopted in the present study and changes in surface pressure and shock detachment distance of a two-dimensional blunt body are obtained by using a perturbation scheme Based on the ratio of the axial distance of a station on the body from the nose (which is the origin of the coordinate frame) to the distance of the "source" location from the origin as a perturbation parameter, first order corrections to the surface pressure and the shock detachment distance are derived.

INTRODUCTION

The plane and axisymmetric hypersonic flow past blunt bodies have been investigated as an inverse problem (shock shape being given) by Chester[1] and Freeman[2]. The fluid is assumed to be in thermodynamic equilibrium locally, and viscosity and heat conduction are neglected. The governing flow equations for a perfect gas are expanded in powers of $(\gamma - 1)/(\gamma + 1)$ and M_∞^{-2}, where γ is the ratio of the specific heats and M_∞ is the Mach number of the free stream. The first approximation corresponding to $\gamma = 1$ and $M_\infty = \infty$ yields a pressure formula identical with a formula given by Busemann[3]. Busemann pressure formula is often referred to as "Newton-Busemann pressure" or "Newtonian-centrifugal pressure." In regions of the flow where pressure is not a small fraction of its value at the nose, the theory of "thin shock layers" of Chester and Freeman yields consistent expressions. However, at some distance downstream of the stagnation point, Freeman's theory becomes invalid as the pressure becomes a small fraction of its value at the nose. A "singularity" appears which, for example, on a sphere will be located at a central half angle of 60°. Schneider's theory[4] removes this difficulty, and provides a more uniformly valid solution. The thin-shock layer assumption is not made in Ref. 4, and changes in velocity along streamlines is permitted. Schneider's theory, thereby, improves Freeman's analysis. (Freeman has also modified his theory to give a uniformly valid solution in the region of the singularity[5].) Freeman's

theory, however, describes the flow conditions near the stagnation region reasonably well. This theory is applied to give the shock detachment distance when the flow is a two-dimensional source-type flow on a two-dimensional blunt body. This condition corresponds to the flow in a wedge shaped nozzle in a hypersonic wind tunnel. The shock stand-off distance increases from its uniform-flow value due to Mach number gradient in the wedge shaped nozzle. The decrement in the surface pressure due to the gradient effect is also obtained by following a perturbation scheme.

ANALYSIS

The flow in a wedge shaped nozzle can be considered to be a source-like flow. The source is assumed to be located near the throat of the nozzle. Any test body in the wind tunnel exposed to a source-like hypersonic flow will experience the effects of the Mach number gradients which arise due to the angularity of the flow. Freeman's theory is extended to obtain the corrections due to the flow angularities. Analysis of the adaption of Freeman's results is described below.

The ideal source flow in two-dimensions leads to the equation

$$\rho_1 q_1 r = \text{const}, \tag{1}$$

where ρ_1 and q_1 are ambient density and velocity respectively. The term r represents the radial distance of the reference station. If the ambient values are assumed to be quantities perturbed about their respective values (indicated by barred quantities) at the nose, we write the equation as

$$(\bar{\rho}_1 + \Delta\rho_1)(\bar{q}_1 + \Delta q_1)(r_o + x') = \bar{\rho}_1 \bar{q}_1 r_o \ . \tag{2}$$

Here r is approximated by $r_o + x'$ where r_o represents the distance of the nose from the throat, and x' is the axial coordinate measured (from the nose) of a reference station on the body. The approximation $r_o + x' \approx r$ is reasonable as we are concerned with a region in the vicinity of the stagnation point. Equation (2) can be written as

$$\frac{\Delta\rho_1}{\bar{\rho}} + \frac{\Delta q_1}{\bar{q}_1} = - \frac{x'}{r_o} \quad , \tag{3}$$

it being assumed that higher powers of x'/r_o are small quantities and are, therefore, negligible. Since in high Mach number flows the relative change in axial velocity, $\Delta q_1/\bar{q}_1$, is small compared to the relative change in density, $\Delta\rho/\bar{\rho}_1$, Eq. (3) becomes

$$\frac{\Delta\rho_1}{\bar{\rho}_1} \simeq -\frac{x'}{r_o} \quad . \tag{4}$$

Also from the relation

$$\frac{\Delta\rho_1}{\bar{\rho}_1} = \frac{\Delta p_1}{\gamma\,\bar{p}_1}$$

we write

$$\frac{\Delta p_1}{\bar{p}_1} = -\frac{\gamma x'}{r_o} \quad . \tag{5}$$

Equations (4) and (5) are now adopted in deriving the corrections due to the ambient flow nonuniformities.

The Newtonian-centrifugal expression for the pressure behind the shock is given by (Ref. 2)

$$\frac{p_{1s}(x,\xi)}{\rho_1 U_1^2} = \sin^2\sigma(x) + \frac{\kappa_o(x)}{k_o(x)} \int_x^{\xi} \Psi'(t)\cos\sigma(t)\,dt \quad . \tag{6}$$

The coordinate system is body oriented, x denoting the curvilinear coordinate along the body surface and y denoting the normal coordinate to the body. The terms in Eq. (6) represent the following quantities:

ρ_1 = ambient density in the nonuniform flow

p_{1s} = pressure in the shock layer at the point of intersection of the streamline ψ = const. and the normal (to the body surface) at x. The reference streamline ψ_s intersects the shock at a point P_s whose body-coordinate is x. Pressure at P_s behind the shock is given by the Rankine-Hugoniot condition

U_1 = ambient velocity in the nonuniform case

$\sigma(x)$ = angle which the ambient-flow-streamline in the nonuniform case makes with the shock at P_s

$\mu(x)$ = angle of inclination of the ambient streamline in the nonuniform flow case with the axis of symmetry

k_o = geometric scale factor which is equal to unity in the two-dimensional case

κ_o = the curvature of the shock (and hence of the body according to the thin shock layer assumption.) This is also given by $-\phi'(x)$ where $\phi(x)$ refers to the body slope (approximately equal to the slope of the shock) at the point x.

Following Freeman's notation, we introduce a quantity $\eta(x)$ which refers to the distance of a point on the body from the line of symmetry. Therefore, $\Phi(t) \simeq \eta(t)$ in the two-dimensional case. In the equations hereafter, we use the subscript "s" to denote quantities behind the shock.

Introducing the perturbation quantities in Eq. (6), we get

$$\frac{\bar{p}_{1s}(x,\xi) + \Delta p_{1s}(x,\xi)}{\rho_1\left(1+\frac{\Delta\rho_1}{\bar{\rho}_1}\right)\bar{U}_1^2\left(1+\frac{\Delta U_1}{\bar{U}_1}\right)^2} = \sin^2(\phi-\mu) + \phi'(x)\int_{\xi}^{x}\eta'(t)\cos(\phi-\mu)dt ,$$

i.e.

$$\frac{p_{1s}(x,\xi)}{\bar{\rho}_1 \bar{U}_1^2}\left[1+\frac{\Delta p_{1s}}{\bar{p}_{1s}}\right]\left[1-\frac{\Delta\rho_1}{\bar{\rho}_1}\right]\left[1-2\frac{\Delta U_1}{\bar{U}_1}\right] \simeq \left[\sin\phi(x)-\mu(x)\cos\phi(x)\right]^2$$

$$+\ \phi'(x)\int_{\xi}^{x}\eta'(t)\ \left[\cos\phi(t)+\mu(t)\sin\phi(t)\right]dt. \qquad (7)$$

In Eq. (7), it is assumed that angle μ is small and $\mu \simeq \eta_x / r_o$. Also powers higher than unity of the relative changes of the flow quantities are neglected as are the cross products of the quantities Δp_{1s}, $\Delta\rho_1$ and ΔU_1. Equation (7) can then be written as

$$\frac{\Delta p_{1s}(x,\xi)}{\bar{\rho}_1 \bar{U}_1^2} = \frac{\Delta\rho_1}{\bar{\rho}_1}\ \frac{\bar{p}_{1s}(x,\xi)}{\bar{\rho}_1 \bar{U}_1^2} + \phi'(x)\int_{\xi}^{x}\eta'(t)\ \mu(t)\sin\phi(t)\,dt$$

$$-\ \mu(x)\sin 2\varphi(x)\ . \qquad (8)$$

We note that $\Delta\rho_1/\bar{\rho}_1 \simeq -x'/r_o$ (x' being the coordinate of the reference point on the body measured along the axis of symmetry from the nose.) Also $\mu(x) = \eta(x)/r_o$.

In Eq. (8), the term $\bar{p}_{1s}$ is given by the expression

$$\frac{\bar{p}_{1s}(x,\xi)}{\bar{\rho}_1 \bar{U}_1^2} = \sin^2\phi(x) + \phi'(x)\int_{\xi}^{x}\eta'(t)\cos\phi(t)\,dt\ .$$

It can be seen that Eq. (8) gives in the vicinity of the blunt stagnation region a surface pressure correction which is approximately one half of the pressure perturbation for a sharp edged body. This is in qualitative agreement with the work of Gordon Hall[6].

The equation for the streamlines in the nonuniform flow case is given by

$$y = \epsilon\int_{\delta}^{\xi}\frac{\sin\phi(t)\sin^2\sigma(t)\,dt}{\left\{\sin^2\sigma(x)+\phi'\int_t^x\sin\phi(t)\cos\sigma(t)\,dt\right\}\left\{\cos^2\sigma(t)-2\epsilon\log\frac{p(x,o)}{\rho_1 U_1^2}\right\}^{1/2}} \qquad (9)$$

where $\epsilon = (\gamma-1)/(\gamma+1)$ (γ being the adiabatic exponent). Further

$$\frac{p(x,o)}{\rho_1 U_1^2} = \sin^2\sigma(x) + \phi' \int_0^x \sin\phi(t)\cos\sigma(t)\,dt . \qquad (10)$$

We note here that in the two-dimensional case

$$\psi'(t) \sim \eta'(t) = \sin\phi(t)$$

After introducing substitutions and approximations as were done earlier, we observe that Eq. (10) becomes

$$\frac{p(x,o)}{\rho_1 U_1^2} = \sin^2\phi(x) - \mu(x)\sin 2\phi(x)$$
$$+ \phi'(x)\left[\int_0^x \sin\phi(s)\cos\phi(s)\,ds + \int_0^x \mu(s)\sin^2\phi(s)ds\right] \qquad (11)$$

We note further that in Eq. (9) the expression

$$R^1(x,t) = \sin^2\sigma(x) + \phi' \int_t^x \sin\phi(s)\cos\sigma(s)\,ds \qquad (12)$$

can be written as

$$R^1(x,t) = \sin^2\phi(x) - \mu(x)\sin 2\phi(x)$$
$$+ \phi'(x)\left[\int_t^x \sin\psi(s)\cos\phi(s)\,ds + \int_t^x \mu(s)\sin^2\phi(s)\,ds\right] . \qquad (13)$$

In view of Eqs. (12) and (13), Eq. (9) can be written as

$$y = \epsilon \int_0^\xi \frac{\left[\sin^3\phi(t) - \mu(t)\sin\varphi(t)\sin 2\phi(t)\right] dt}{R^1(x,t)\left[\cos^2\phi(t) + \mu\sin 2\varphi(t) - 2\epsilon\log R^1(x\ o)\right]^{1/2}} . \qquad (14)$$

The equation for the shock is obtained by replacing ξ by x. We are now able to evaluate the shock "stand-off" distance in the two-dimensional case. We note that for small values of t, $\eta(t) \sim t$, and $\phi \simeq \Pi/2 - t/a$, where a is the radius of curvature of the nose. With these approximations,

Eq. (14) becomes

$$y = \epsilon \int_0^{x} \frac{\left[\cos^3 \frac{t}{a} - \frac{t}{r_o} \cos\frac{t}{a} \sin\frac{2t}{a}\right] dt}{R^1(x,t) \left[\sin^2\frac{t}{a} + \frac{t}{r_o} \sin\frac{2t}{a} - 2\epsilon \log R^1(x,o)\right]^{1/2}} \tag{15}$$

where $\mu(x) \simeq \eta_x / r_o \sim x / r_o$ when x is small,

and

$$R^1(x,t) = \frac{3}{2}\cos^2\frac{x}{a} - \frac{1}{2}\cos^2\frac{t}{a} - \frac{x}{r_o}\sin\frac{2x}{a} - \frac{1}{4ar_o}(x^2 - t^2)$$

$$- \frac{a}{8r_o}\left[\frac{2x}{a}\sin\frac{2x}{a} + \cos\frac{2x}{a} - \frac{2t}{a}\sin\frac{2t}{a} - \sin\frac{2t}{a}\right] .$$

After some algebra, Eq. (15) can be written as

$$y = \epsilon \int_0^x \frac{\left(\cos^3\frac{t}{a}\right) dt}{D} + \frac{\epsilon}{r_o} \int_0^x \frac{F(x,t)\, dt}{D} \tag{16}$$

where

$$D = \left(\frac{3}{2}\cos^2\frac{x}{a} - \frac{1}{2}\cos^2\frac{t}{a}\right)\left(\sin^2\frac{t}{a} + 2\epsilon\log\frac{2}{\left(3\cos^2\frac{x}{a} - 1\right)}\right)^{1/2}$$

and

$$F = \frac{2\cos^3\frac{t}{a}}{\left(3\cos^2\frac{x}{a} - \cos^2\frac{t}{a}\right)}\left[\frac{x^2}{4a} + \frac{5x}{4}\sin\frac{2x}{a} + \frac{a}{8}\cos\frac{2x}{a} - \frac{t^2}{4a} - \frac{t}{4}\sin\frac{2t}{a} - \frac{a}{8}\sin\frac{2t}{a}\right]$$

$$- \frac{t\sin\frac{t}{a}\cos^4\frac{t}{a}}{\left(\sin^2\frac{t}{a} + 2\epsilon\log\frac{2}{\left(3\cos^2\frac{x}{a} - 1\right)}\right)} - 2t\sin\frac{t}{a}\cos^2\frac{t}{a} .$$

The first integral on the right hand side of Eq. (16) can easily be evaluated, and in the limit as $x \to 0$, the value of the integral (which is $a\epsilon/2 \log(4/3\epsilon)$) gives the shock stand-off distance in the uniform-flow case. The second

integral on the right hand side of Eq. (16) is the correction to the stand-off distance due to the ambient flow nonuniformities. The integral can be evaluated in a straight forward manner, and the value, in the limit as $x \to 0$, becomes

$$\Delta\delta = \frac{a^2}{16 r_o} \epsilon \log \frac{4}{3\epsilon} \quad .$$

We can, therefore, write the shock stand-off distance in the nonuniform case as

$$\frac{\delta'}{a} = \frac{\delta}{a} + \frac{\Delta\delta}{a} \tag{17}$$

where

$$\frac{\delta}{a} = \frac{\epsilon}{2} \log \frac{4}{3\epsilon} \qquad \text{(in the uniform flow case)} \tag{18}$$

and

$$\frac{\Delta\delta}{a} = \frac{1}{8} \frac{a}{r_o} \left(\frac{\delta}{a}\right) \quad , \text{(correction due to the ambient flow nonuniformities)} \tag{19}$$

Equation (19) gives the increment in the shock stand-off distance due to angular divergence of the ambient flow. The increment can be quite significant when the radius of curvature at the nose of the test body is large.

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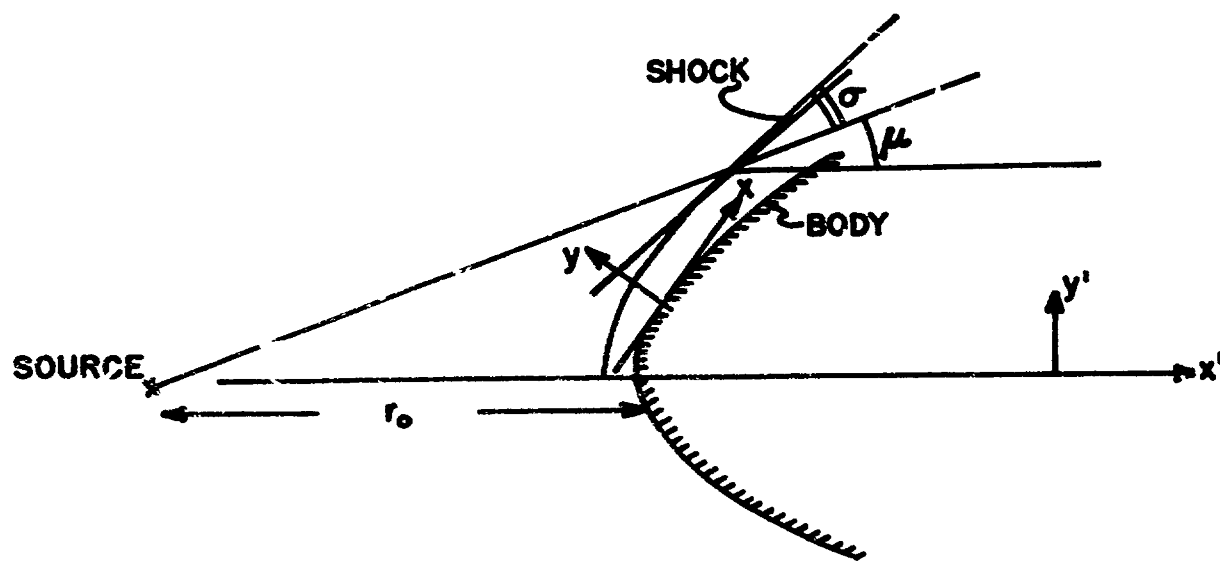

FIG. I CO-ORDINATE SYSTEM IN THE ANALYSIS

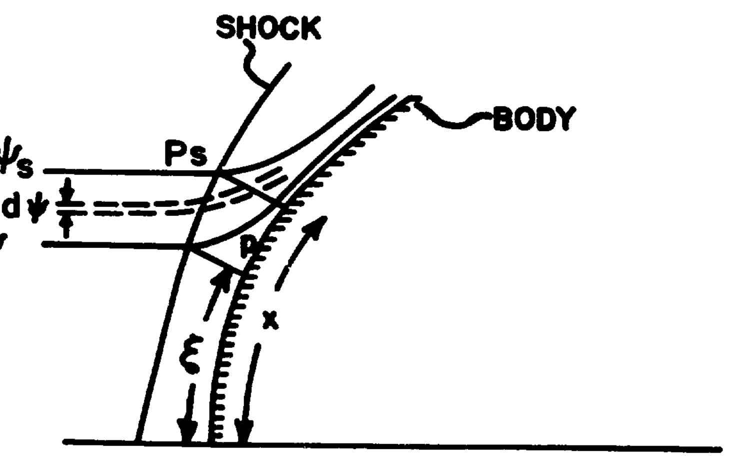

FIG. 2 STREAMLINES IN THE FLOW FIELD